A State Space Model for Vibration Based Prognostics

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ABSTRACT

Operation and maintenance of offshore wind farms will be more difficult and expensive than equivalent onshore wind farms. Accessibility for routine servicing and maintenance will be a concern: there may be times when the offshore wind farm is inaccessible due to sea, wind and visibility conditions. Additionally, maintenance tasks are more expensive than onshore due to: distance of the offshore wind farm from shore, site exposure, and the need for specialized lifting equipment to install and change out major components.

As a result, the requirement for remote monitoring and condition based maintenance techniques becomes more important to maintain appropriate turbine availability levels. The development of a prognostics health management (PHM) capability will allow a strategy that balances risk of running the turbine versus lost revenue. Prognostics would give an estimate of the remaining useful life of a component under various loads, thus avoid component failure.

We present a state space model for predicting the remaining useful life of a component based on vibration signatures. The model dynamics are explained and analysis is performed to evaluate the nature of fault signature distribution, and an indicator of prognostic confidence is proposed. The model is then validated under real world conditions.*

1 Introduction

It is anticipated that 20% of the United States electrical energy will come from wind power by 2020. A larger percentage of that power will come from offshore wind turbines. Current offshore wind turbine initial capital costs are 30-50% higher than onshore. These costs are offset by higher capacity factors (anticipated at 31%) and higher mean wind speeds resulting in higher energy yields (as much at 30%, Meyer, 2009).

Offshore power will have its challenges. Operations and Maintenance cost will be higher. These locations will be more remote than on shore systems. Additionally, they will typically be larger, requiring purpose built lifting equipment to install and change out major component. Further, there will be times when the offshore turbine in not accessible for maintenance due to poor weather conditions. This will require remote sensing and condition based maintenance techniques for diagnostics and prognostic, e.g. the development of a PHM system (Kuhn et al., 1997).

Diagnostics is concerned with detection of a changing condition related to a component failure. Once a changing condition is detected, prognosis provides the time to failure (remaining useful life or RUL) and an associated confidence in that prediction (Vachtsevanos et al. 2006). This information, RUL and confidence, is used in different way by the maintainer and operator.

In general, prognostics will allow offshore turbines to be operated at lower cost by:

- Opportunistic maintenance practices
- Improved Readiness/Reduction in unscheduled maintenance

- Activation of a "just in time" part delivery,
- A reduction in the overall number of spares
- Operating the turbine in a conservative manner to reduce the chance of failure. By operating under less stressful loads, extend the life of a component until maintenance can be performed.

2 STATE SPACE MODELS FOR PROGNOSTICS

State space representation of data provides a versatile and robust why to model systems. We start with the definition of the states, and the basic principles underlying the characterization of phenomena under study. Then we show how the states propagate as a stochastic process.

The choice of which type of state space model to use is driven by the nature of the system dynamics and noise source. If linear dynamic, with Gaussian noise, a Kalman filter (KF) is used. If it is a non-linear process, with Guassian noise, a sigma-point Bayesian process (e.g. unscented Kalman filter - UKF) or extended Kalman filter (EKF) is appropriate. For non-linear dynamics with non-linear noise, a sequential Monte Carlo method employing sequential estimation of the probability distribution using "importance sampling" techniques is used. This method is generally referred to as particle filtering (PF) (Candy 2009).

2.1 A State Space Model

A state space model estimates the state variable on the basis of measurement of the output and input control variables (Brogan 1991). In general, a system plant can be defined by:

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u}$$
$$\mathbf{y} = \mathbf{C}\mathbf{x} \qquad (1)$$

where $\mathbf{x}$ is the state variable, $\dot{\mathbf{x}}$ is the rate of change of the state variable, and y is the output of the system.

An observer is a subsystem used to reconstruct the state space of the plant. The model of the observer is the same as that of the plant, expect that we add an additional term which includes the estimated error to account for inaccuracies the **A** and **B** matrixes. This means that any hidden state (such as RUL) can be reconstructed if we can model the plant (e.g. failure propagation) successfully. The observer is defined as:

$$\dot{\hat{\mathbf{x}}} = \mathbf{A}\hat{\mathbf{x}} + \mathbf{B}\mathbf{u} + \mathbf{K}(\mathbf{y} - \mathbf{C}\hat{\mathbf{x}}) \qquad (2)$$

where $\dot{\hat{\mathbf{x}}}$ is the estimate state and $\mathbf{C}\hat{\mathbf{x}}$ is the estimated output. The matrix **K** is called the Kalman gain matrix (linear, Gaussian case), it is a weighting matrix that maps the differences between the measured output **y** and the estimated output $\mathbf{C}\hat{\mathbf{x}}$. A KF can be used to optimally set the Kalman Gain matrix. Figure 1 represents a system and its full state observer.

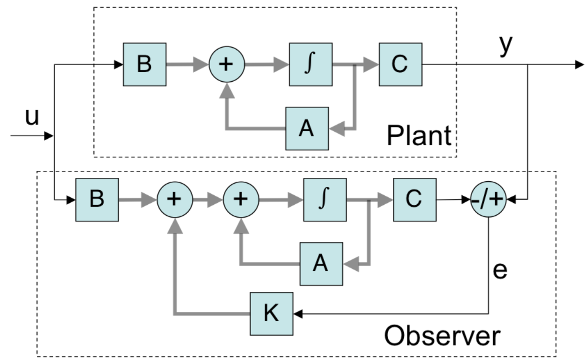

Figure 1 Example of Plant and State Observer

2.2 The General Case: Kalman Filter

A KF is a recursive algorithm that optimally filters the measured state based on *a priori* information such as the measurement noise, the unknown behavior of the state, and relationship between the input and output states (e.g. the plant), and the time between measurements. Computationally it is attractive because it can be designed with no matrix inversion and it is a one step, iterative process. The filtering process is given as:

Prediction

$X_{t|t-1} = A\,X_{t-1|t-1}$ State

$P_{t|t-1} = A\,P_{t-1|t-1}A' + Q$ Covariance

Gain

$K = P_{t|t-1}\,C'\,[C\,P_{t|t-1}\,C' + R]^{-1}$

Update

$P_{t|t} = (I - KC)\,P_{t|t-1}$ State Covariance

$X_{t|t} = X_{t|t-1} + K(Y\text{-}H\,X_{t|t-1})$ State Update

where:

t|t-1	is the condition statement (e.g. t given the information at t-1)
X	is the state information (x, xdot, x dot dot)
Y	is the measured data
K	is the Kalman Gain
P	is the state covariance matrix
Q	is the process noise model
C	is the measurement matrix
R	is the measurement variance

For nonlinear systems with Gaussian noise (UKF or EKF), the state prediction is now a function of $X_{t-1|t-1}$, **A** and **C** is the Jacobian (e.g the derivative of the measurement with respect to time) (Candy 2009).

For non-linear, non-Gaussian noise problems, particle filters (PF) are attractive. PF is based on representing the filtering distribution as a set of particles. The particles are generated using sequential importance re-sampling (a Monte Carlo technique), where a proposed distribution is used to approximate a posterior distribution by appropriate weighting.

An important consideration for Monte Carlo methods (such as PF) is that it requires an estimate of the posterior distribution using sample based simulation (Candy 2009). Starting with Bayes rules:

$$\Pr(X \mid Y) = \Pr(Y \mid X) * \Pr(X) / \Pr(Y) \quad (3)$$

where X is the distribution of the measurement, and Y is the resampled distribution, we can see that the performance is heavily conditioned on the selection of measurement PDF (distribution, first and second moment). However, this estimation of the X for the linear (KF) and non-linear (EKF) case is also important, as it determines the R the measurement noise model.

For all models, the best estimate for the first moment is the state-space model itself. It is assumed that one of the states of interest is the expected value of the measurement. In general, the second moment (variance: σ^2) is assumed constant. This is a poor assumption. In this study, an accessory calculation of variance was made using a recursive estimate:

$$\sigma_t^2 = (1-a)\sigma_{t-1}^2 + \left(CE\left[X_{t|t-1} \mid Y_{t-1}\right] - Y\right)^2 \quad (4)$$

Using a Butterworth filter design and a normalized bandwidth of 0.1, a is given as 0.2677. The implementation of the EKF and PF was done utilizing the fine Rao-Blackwellized particle filters Matlab toolbox from J Hartikainen and S. Sarkka (Hartikainen and Sarkka 2008).

3 SYSTEM DYNAMICS

The state space model can be constructed as a parallel system to the plant (e.g. the system under study). This requires an appropriate model to simulate the system dynamics. In general, failure modes propagating in mechanical systems are difficult to model at a level of fidelity that would generate any meaningful results (e.g. Health and RUL). We needed a generalized, data driven process that would model the plant adequately enough to generate RUL with small error.

Since 1953, a number of fault growth theories have been proposed, such as: net area stress theories, accumulated strain hypothesis, dislocation theories, and others (Frost et al 1999, Frost 1959). Through substitution of variables, many of these theories can be generalized by the Paris Law:

$$da/dN = D(\Delta K)^m \quad (5)$$

which governs the rate of crack growth in a homogenous material, where:

da/dN is the rate of change of the half crack length
D is a material constant of the crack growth equation
ΔK is the range of the K during a fatigue cycle
m is the exponent of the crack growth equation

The range of strain, ΔK is given as:

$$\Delta K = 2\sigma\alpha(\pi a)^{1/2} \quad (5)$$

here

σ is gross strain
α is a geometric correction factor
a is the half crack length

The use of Paris's law for the calculation of RUL was given by (Bechhoefer 2008) and (Orchard et al. 2007), but lacking a measure of confidence (e.g. how good is the prognostics). Confidence, or a measure of how good the model is, is a requirement for a PHM system (Vachtsevanos et al., 2006).

These variables are specific to a given material and test article. In practice, the variables are unknown. This requires some simplifying assumptions to be made to facilitate analysis. For many components/material, the crack growth exponent is 2. The geometric correction factor α, is set to 1, which allows equation (4) to be reduced to:

$$da/dN = D(4\sigma^2\pi a) \quad (6)$$

The goal is to determine the number of cycles, N, remaining until a crack length a is reached. Taking the reciprocal of (6) gives:

$$dN/da = 1/D(4\sigma^2\pi a) \quad (7)$$

Integrating gives the number of cycles (N) remaining. Note that N for synchronous systems (e.g. constant RPM) is equivalent to time by multiplying with a constant.

$$\begin{aligned} N &= \int_{a_o}^{a_f} dN/da \\ &= \int 1/D(4\sigma^2\pi a)\, da \\ &= 1/D(4\sigma^2\pi)\left(\ln(a_f) - \ln(a_o)\right) \end{aligned} \quad (8)$$

Equation (8) gives the number of cycles N from the current measured crack a_o to the final crack length a_f. The measured component condition indicator (CI) will be used as a surrogate for crack length a. Given a suitable threshold set for a_f (Bechhoefer and Bernhard 2007) then N is the RUL times some constant (RPM for a synchronous system).

The material crack constant, D, can be estimated as:

$$D = da/dN * (4\sigma^2\pi a) \quad (9)$$

In practice gross strain will not be know. Again, a surrogate value, such as torque, will be as appropriate.

3.1 Prognostic and Confidence in the Prognostics

In practice, a prognostic or PHM capability would be used to schedule maintenance or assist in assets management. Maintainers and operators will perform management of the offshore assets. They will need an intuitive, simple display that conveys information on: current health, RUL, and confidence in the RUL prediction.

Model confidence is essential in any RUL prediction (Vachtsevanos et al., 2006). For any RUL calculation, given 1 hour of nominal usage, the RUL should decrease by 1 (e.g. dN/dt is approximately -1: on hour of life is consumed). Further, a measure of model drift or convergence is the second derivative d^2N/dt^2: a value close to zero indicates convergences. When these conditions are met, the model used for calculation of the RUL is consistent, and is indicative of a good estimate of the life of the component.

We will use visual cues for of the prognostics based on model convergence. The prognostic color reflects the confidence:

- Low Confidence: Yellow, abs(dN/dt-1) > 3 and abs(d^2N/dt^2) > 0.5
- Medium Confidence: Blue abs(dN/dt-1) > 2 and abs(d^2N/dt^2) > 0.5
- High Confidence: Green, abs(dN/dt-1) < 2 and abs(d^2N/dt^2) < 0.5

Another aspect of the prognostic model is to predict what the health of the component will be some time in the future. For a given state space mode, the RUL or any predicted health is an expectation based on the current state and future usage (e.g. damage or stain). The Paris law is driven by delta strain: changes in strain will affect the RUL (eq 5). Future health is then based on the mean strain, and a bound on that strain. This strain information could be based on forecast weather or usage for a wind turbine. The health at any time in the future is then:

$$a_f = \exp\left(ND\left(4\sigma^2\pi\right) + \ln\left(a_o\right)\right) \qquad (10)$$

The upper and lower bound on the future health a_f can then be calculated through bounding the delta strain (e.g. 5% and 95% value of delta strain).

3.2 Normality of Plant/System Noise

Selection of the appropriate state space model is dependent not only on the system dynamics, but the measurement noise of the system. We investigated the probability density function of the measurement noise by using a kernel smoothing density estimate of 100 data points (49 prior and 50 after a point of interest) and compared this to a Gaussian distribution with mean and standard deviation of the kernel (figure 2).

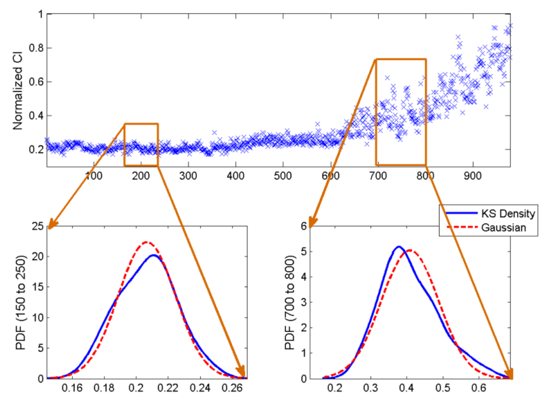

Figure 2 Estimate of PDF of the measurement noise.

For example, figure 2 displays the data (signal average RMS data associated with a hydraulic pump), and estimates of the measurement PDF at point 200 (prior to fault propagation) and at point 750 (well into fault propagation). The PDF estimate is plotted against the Gaussian PDF based on the mean and standard deviation of the kernel. Even given that this data is inherently non-stationary, the kernel estimate is close to Gaussian. For this reason (non-linear dynamics, Gaussian noise), an EKF was used for the state space model.

4 DEMONSTRATION OF PHM

For this application of a state space observer, it was determined that the system dynamics were nonlinear, and that the measurement noise was Gaussian. For this reason, and EKF model was constructed. The states of the model are:

- a: half crack length with is linear with the measurement.
- da/dN: rate of change of half crack length with respect to the number of cycles, which is linear with time.
- N: the number of cycles remaining, which is linear with time (synchronous system) and is non-linear with respect to half crack length (nature log).
- dN/dt: rate of change in number of cycles with respect to time, which is linear with time
- d^2N/dt^2: the acceleration of the number of cycles, which is linear with time.

A five state model was developed, with an accessory estimation of measurement noise made using a recursive estimate of measurement variance (eq 4). The velocity and acceleration of N was used to define confidence in the estimate. Error bounds for the prognostics were based on the 5% and 95% of load, and 5% and 95% of bound on the estimate of *a*.

4.1 PHM Test Article

As a result of several helicopter accidents in the 1990s, the United Kingdom Civil Aviation Authority mandated vibration health monitoring (VHM) systems in June of 1999 (CAP753, 2006). As a result, the helicopter industry has matured this technology faster than in the industrial market. The helicopter industry has close to 30 years development in VHM, with over 1200-installed systems: a large database of faults is being developed.

There are a number of similarities between helicopter transmissions and wind turbines: similar gear ratios (80:1) and through put power. As such, we are demonstrating prognostics with the hydraulic pump on a utility helicopter. This component is driven at constant RPM by an auxiliary gearbox off of the drive train gearbox. The helicopter has a VHM system that generates condition indicators (CIs) associated with the hydraulic pump drive shaft, but was not configured to measure health of the hydraulic pump. While reviewing VHM data, it was observed that the time synchronous average (TSA) RMS where trending upward. The shaft order 1, 2 and 3 values (which give indications of shaft condition) where nominal. Along with CI values, raw time domain data was also collected on this shaft. Analysis of this time domain data showed that the elevated TSA RMS was driven exclusively by a 9 per rev (acceleration corresponding to 9 time the shaft RPM), which is associated with the 9 piston hydraulic pump driven by this shaft. The peak-to-peak acceleration of the pump was seen to be up to 30 Gs' prior to failure.

Because there are no vibration-based limits applied to this component, we can set a limit that would be appropriate for industrial monitoring, such as .75 inch per second (ips) peak-to-peak. Given the shaft operating speed of 11,806 RPM, the conversion from RMS to ips peak-to-peak is:

$$HI = TSA\ RMS * 32.174\, ft/\sec^2 * 12 in/ft\, /(2\pi) *$$
$$60\sec/\min/11806\text{rpm} * \sqrt{2}\,/9 rev$$
$$= TSA\ RMS * 0.049$$

Figure 3 displays the raw pump and state observer health vs. flight hours.

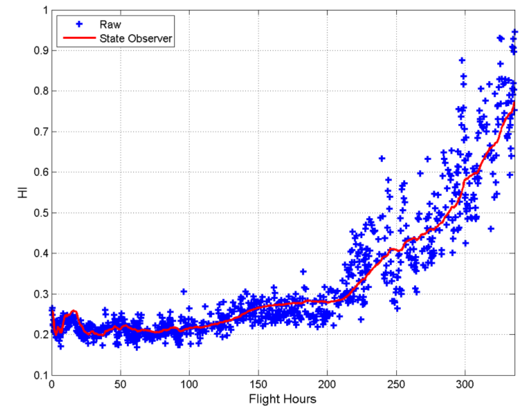

Figure 3 Raw CI and State Space for CI

Figure 4 is the RUL of the pump. Prior to time 100, the RUL is effectively infinite because *dH/dt* is close to zero. At time 120, corresponding to an increase the in pump HI value, the RUL decreases rapidly. At time 270 (55 remaining flight hours) the estimated RUL tracks with the actual RUL

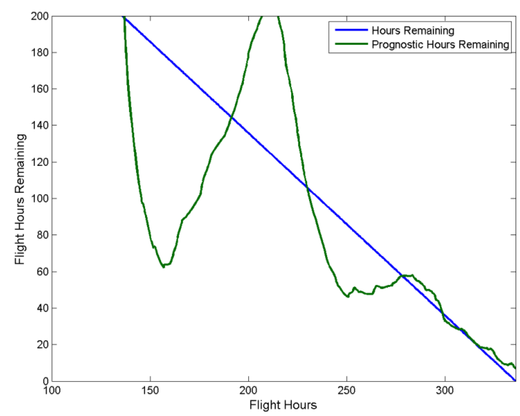

Figure 4 Pump Actual and Estimated RUL

The derivative of the RUL is given in figure 5. An automated maintenance action could be triggered based on the dRUL/dt is close to -1, reporting hours remaining.

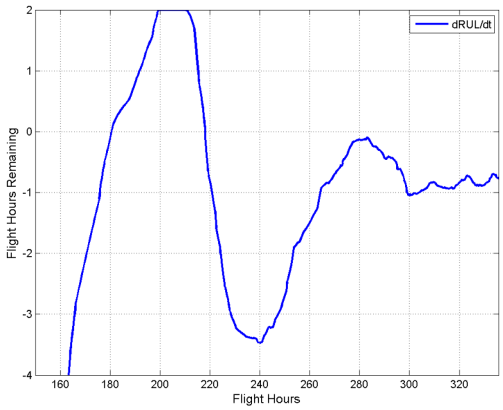

Figure 5 Derivative of Estimated RUL

The derivative converges to a value of -1 at 270 to 300 hours (pump failed at 335 hour). Consider that the CI is not a direct measure of any feature on the hydraulic pump. With a CI directly measuring SO9, one could assume that the prognostics capability would be increased.

4.2 Confidence in the Prognostics

In practice, a PHM system would be used to schedule maintenance or assist is assets management. The Maintainer or Operator will need an intuitive, simple display that conveys information on: current health, RUL, and confidence in the RUL prediction. The state space model contains all of the information needed to support a PHM process, by:

- Giving a bound on error. The state space covariance and current state health is used to bound the RUL by 5% to 95% and to project he expected condition of the component out into the future.
- The state space model fit (e.g. *dRUL/dt* = -1) is used to validate the model and give a visual cue that the prognostic has value.

Using state space model, the component condition is plotted over a window of 100 hours of past history to 100 hours into the future (this length scale is based on the logistic timeline of the component. For example, 100 hours of flight time is approximately 1 month. For wind turbines, 2000 hours may be more appropriate).

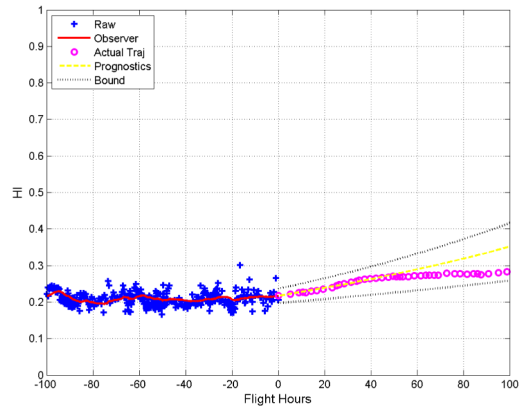

Figure 6 Prognostics with Error Bounds at time 107 Hours

Figure 6 displays the hydraulic pump health at time 107 hours, as the component fault just starts to propagate. Figure 7 shows the state space model at time 204 hours. The model confidence has improved (blue line) indicating that dRUL/dt is approaching -1. The bounds on error have increased, reflecting increased measurement noise of the system.

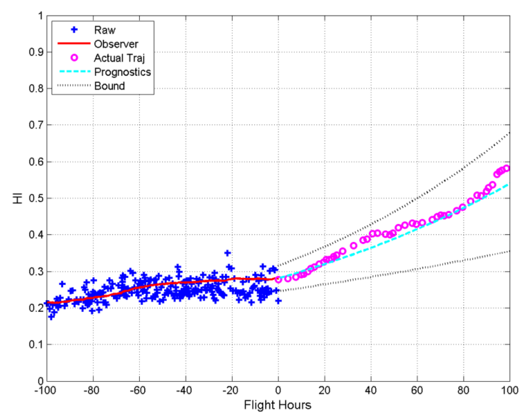

Figure 7 Prognostics with Error Bounds at time 204 Hours

Figure 8 shows the prognostic at time 284 hours, with approximately 55 hours of remaining life. The prognostic suggests that if the component was operated at a lower power setting (e.g. at 45% torque vs. a nominal 60% torque) the component life could be extended to 100 hours.

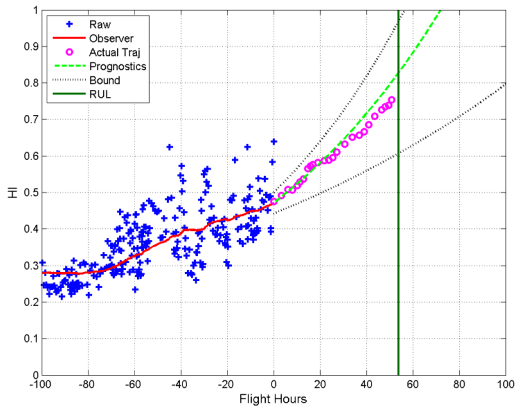

Figure 8 Prognostics with Error Bounds at time 284 Hours

The ability to see the effect of operating condition on the life of a part is a powerful management tool. Note the prognostic is remarkably close to the actual fault propagation trajectory once the model has converged.

The ability to reconstruct the damage or component state and to estimate RUL using the Paris' Law fault propagation model is robust. It requires only one *a priori* configuration item: a value for plant noise. Equally important to the PHM system is a means to quantify model fit and confidence, which is conveniently calculated from the first derivative of RUL.

An explanation as to why the model did not converge until 270 flight hours could be a result of the CI not directly measuring SO9. Initially, the TSA RMS is driven by shaft order 1, 2, 3, and a 92/rev spur gear. The performance likely would be improved using only SO9 amplitude data. At some future point, this may become a new CI for this component, which would result in a better model fit and a corresponding increase in prognostic capability.

5 CONCLUSIONS

Evaluation of component health or the ability to predict incipient failure remains difficult. State space modeling may be shown to facilitate the maturation of PHM system to allow robust condition monitoring and prognostics capability. In this paper, we apply the concept of a state space model, taken from control theory, as tool for developing a PHM system.

Presented are techniques taken from of control theory to "observe" hidden states associated with component health. This technique has been show to work with high speed input shafts, bearings (real world and test stand see (Bechhoefer 2008)) and complex device, such as the hydraulic pump. It is anticipated that similar performance would be observed with gear failure.

The power of the techniques is in the generality of the approach and the ability to successfully determine the remaining useful life with limited data. In one example, only signal average RMS from a hydraulic pump is used to predict failure 55 flight hours in the future. While this technique appears a step closer to achieving reliable prediction of remaining useful life, addition work needs to be done to implement in a system. The next goal would be to implement in an actual PHM on a wind turbine.

Over the next two years, we will work toward engaging a customer to demonstrate this capability on a wind turbine.

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